Selling Shares to Budget-Constrained Bidders: An Experimental Study of the Proportional Auction

(J. Bae and J. H. Kagel)

Appendix

A.1 Uniqueness of the equilibrium in PA with a budget constraint.

Let $\boldsymbol{b} = \{b_i, \boldsymbol{b}_{-\boldsymbol{i}}\}$ be the bid profile and $x_i(\boldsymbol{b}) = \frac{b_i}{b_i + \sum_{-i} \boldsymbol{b}_{-\boldsymbol{i}}}$ the share bidder i gets. Player i's payoff is

$$\Pi(\boldsymbol{b}; v_i) = x_i(\boldsymbol{b})v_i - b_i h(b_i)$$

Where

$$h(b_i) = \begin{cases} 1 & \text{if} \quad b_i \le w \\ \infty & \text{if} \quad b_i > w \end{cases}$$

note that $h(b_i)$ integrates the budget constraint into the payoff.

Wasser (2013) showed that, with a proper transformation of variables, the maximization problem of the payoff function is equivalent to the maximization problem of the following payoff function.

$$\widetilde{\Pi}(\boldsymbol{b}; c_i) = x_i(\boldsymbol{b}) - c_i b_i h(b_i)$$

Since no bidder would submit bids higher than w, we can confine our interest to the interval where $0 \le b_i \le w$ and $h(b_i) = 1$. Then the existence and uniqueness of the equilibrium directly follows from the results of Ewerhart (2014).

A.2 Comparative statics between FPA and PA with increasing number of bidders

Figure A.2. Efficiency and revenue predictions with changing numbers of bidders

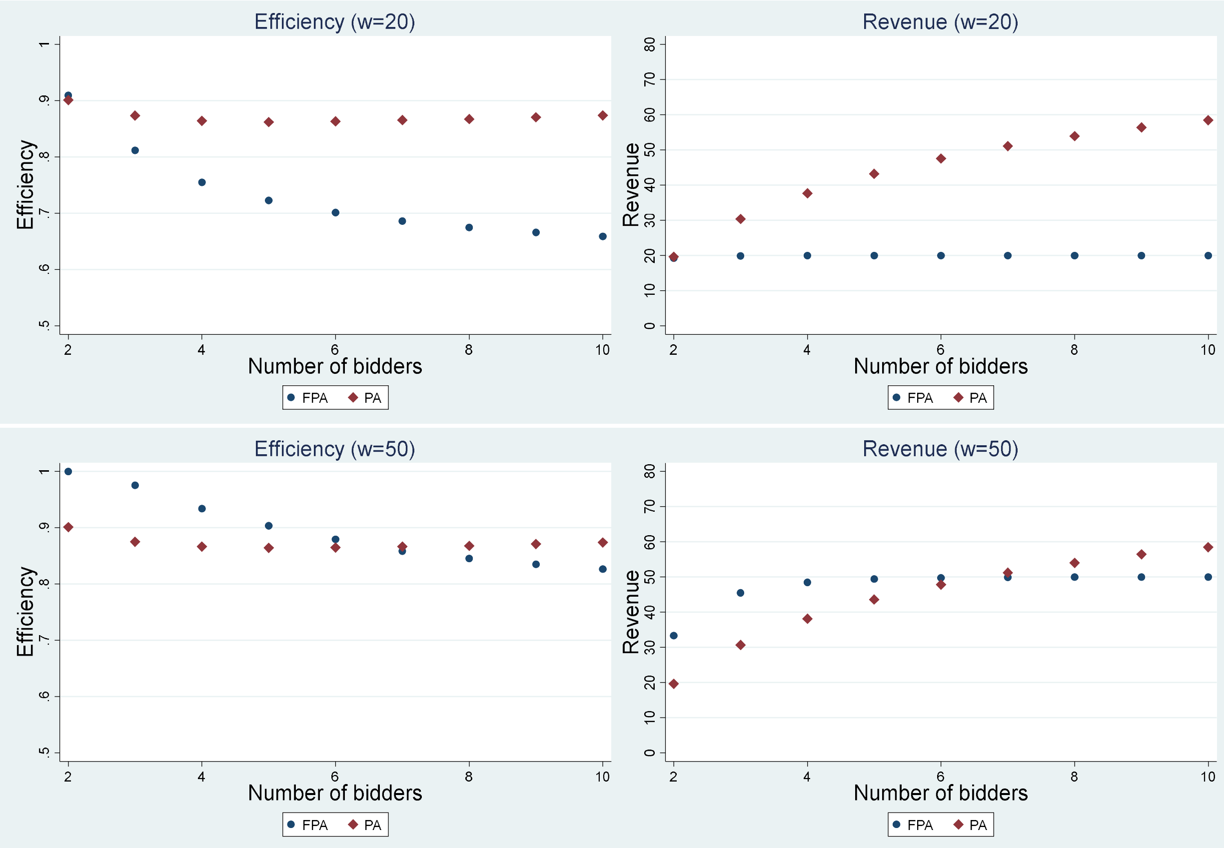

Figure A.2 shows efficiency and revenue predictions when the number of bidders changes. When the budget is tight (w=20, upper panel), PA achieves higher efficiency and revenue than FPA, with the gap between the two increasing with more bidders. As the number of bidders increases, the highest bidder value is likely to increase, so that the maximum possible surplus gets bigger. However, FPA essentially randomly assigns items regardless of bidder values since most bidders' pool at the budget constraint. So the realized surplus does not improve with the number of bidders. Thus, the efficiency of FPA decreases as the number of bidders increases.

When the budget is weaker (w=50, lower panel), FPA is predicted to achieve higher efficiency and revenue than PA if the number of bidders is small ($n \leq 6$). However, when the number of bidders increases ($n > 6$), the efficiency and revenue of PA surpasses FPA, with this gap increasing as the number of bidders increases. So that PA can have higher revenue and efficiency than FPA under a weak budget as the number of bidders increases.

A3. Regression analysis for efficiency and revenue

Table A.1. Regression results with PA dummy variable

VARIABLES	Efficiency (w=20)	Efficiency (w=50)	Revenue (w=20)	Revenue (w=50)
Period	-0.004	0.000	0.020	0.185
	(0.002)	(0.002)	(0.087)	(0.256)
PA	0.054***	-0.065***	9.909***	-8.840***
	(0.009)	(0.007)	(0.517)	(0.318)
Constant	0.836***	0.960***	19.644***	45.574***
	(0.011)	(0.014)	(0.486)	(1.452)
Observations	390	390	390	390
R-squared	0.023	0.100	0.262	0.078

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

This table reports regression results for efficiency and revenue for PA compared to FPA with clustering standard errors at session level. The omitted variable is FPA. The negligible period values show that auction outcomes were stable over time.

Under w=20, efficiency in PA was 5.4 percentage points higher than in FPA, and revenue was 9.9 ECUs (experimental currency units) higher than in FPA. Under w=50, efficiency was 6.5 percentage points lower under PA, and 8.8 ECUs lower revenue than FPA.

A4. Dominated bids in PA

Proposition 2. In PA, bidding more than $1/4v_i$ is dominated by bidding $1/4v_i$, regardless of n and $F(v)$.

Proof. Let $\boldsymbol{b}_{-i} = (b_1, b_2, \dots b_{i-1}, b_{i+1}, \dots b_n)$ be bids submitted by bidders except bidder i. Bidder i's payoff of bidding b_i given v_i is

$$\Pi(b_i, \boldsymbol{b}_{-i}, v_i) = \frac{b_i}{b_i + \sum_{-i} b_{-i}} v_i - b_i$$

The marginal payoff is

$$\frac{\partial \Pi(b_i, \boldsymbol{b}_{-i}, v_i)}{\partial b_i} = \frac{\sum_{-i} b_{-i}}{(b_i + \sum_{-i} b_{-i})^2} v_i - 1$$

I will show that if $b_i > 1/4v_i$, the marginal payoff is always negative regardless of $\boldsymbol{b}_{-i}$. This implies that payoff of bidding any bid $b_i > 1/4v_i$ is smaller than bidding $b_i = 1/4v_i$

$$\frac{\sum_{-i} b_{-i}}{(b_i + \sum_{-i} b_{-i})^2} v_i - 1 \leq \frac{b_i}{(b_i + b_i)^2} v_i - 1 = \frac{1}{4b_i} v_i - 1 < 0$$

The first inequality is holds since the first term is maximized when $\sum_{-i} b_{-i} = b_i$. The last inequality holds since $b_i > 1/4v_i$.

A5. Dominated strategy in PA regardless of risk attitude

Proposition 2*. In PA, bidding more than $1/4v_i$ is dominated by bidding $1/4v_i$, regardless of n and $F(v)$. This holds with any risk attitude.

Proof. Let $\boldsymbol{b}_{-i} = (b_1, b_2, \ldots b_{i-1}, b_{i+1}, \ldots b_n)$ be bids submitted by bidders except bidder i and $u(c)$ be the utility function with $u'(c) > 0$.

Bidder i's payoff of bidding b_i given v_i is

$$\Pi(b_i, \boldsymbol{b}_{-i}, v_i) = u\left(\frac{b_i}{b_i + \sum_{-i} b_{-i}} v_i - b_i\right)$$

The marginal payoff is

$$\frac{\partial \Pi(b_i, \boldsymbol{b}_{-i}, v_i)}{\partial b_i} = u'\left(\frac{b_i}{b_i + \sum_{-i} b_{-i}} v_i - b_i\right)\left(\frac{\sum_{-i} b_{-i}}{(b_i + \sum_{-i} b_{-i})^2} v_i - 1\right)$$

Since others' bids affect bidder *i's* s payoff only through the summation, let $\sum_{-i} b_{-i} = x$ for notational convenience. And denote $\frac{b_i}{b_i + \sum_{-i} b_{-i}} v_i - b_i = c$. I will show that if $b_i > 1/4v_i$, the marginal payoff is always negative for any x, so that bidding above $1/4v_i$ is always inferior to bidding $1/4v_i$.

$$u'(c)\left(\frac{x}{(b_i + x)^2} v_i - 1\right) \leq u'(c)\left(\frac{b_i}{(b_i + b_i)^2} v_i - 1\right) = u'(c)\left(\frac{1}{4b_i} v_i - 1\right) < 0$$

Note that $u'(c) > 0$. The first inequality is holds since $\frac{x}{(b_i+x)^2} v_i$ is maximized when $x = b_i$. The last inequality holds since $b_i > 1/4v_i$

A.6 Reverse order sessions

Below are results for three reverse order sessions starting with w = 50 and crossing over to w = 20. There are 2 FPA sessions and 1 PA session. The pandemic resulting in closing the experimental economics laboratory before more sessions could be completed. However, the results are quite clear – reversing the order from a weak to a strong budget constraint does not change the results.

Revenue and Efficiency

	Budget	Predicted		Realized	
		FPA	PA	FPA	PA
Efficiency	*w*=20	80.4% (1.1%)	87.4% (0.7%)	78.7% (2.2%)	86.1% (1.1%)
	w=50	94.1% (1.4%)	87.6% (0.7%)	94.1% (1.4%)	88.2% (0.9%)
Revenue	*w*=20	19.9 (0.08)	29.0 (1.62)	19.9 (0.08)	30.4 (1.73)
	w=50	46.1 (0.81)	29.3 (1.67)	46.4 (0.76)	38.2 (2.60)

Num of obs = 240 for FPA-Rev
Num of obs = 120 for PA-Rev