Selling Shares to Budget-Constrained Bidders: An Experimental Study of the Proportional Auction

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Abstract

This paper explores experimentally the efficiency and revenue of a proportional auction (PA) compared to a first price auction (FPA)for budget constrained bidders. PA auctions have been used in privatization of Russian assets and in cryptocurrency sales as they can achieve higher efficiency and revenue with budget constrained bidders than in an FPA. Experimental results show that consistent with theoretical predictions, under a strong budget constraint PA achieved higher revenue and efficiency than FPA, with these results reversed under a looser budget constraint. Detailed patterns of bidding are compared to theoretical predictions for both PA and FPA.

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1. Introduction

Many auctions sell items that could be sold in parts: shares of a company, mineral rights, electricity from a generator, and shares of facilities. If bidders are willing and able to buy the entire amount offered, the single-unit first price ich the highest bidder wins the whole item, allocates the item efficiently and raises the most revenue (Myerson, 1981). However, when bidders' budgets constrain buying the entire item, a *proportional auction* (PA) can be more profitable and achieve higher efficiency than the first price auction (FPA). In PA each bidder submits a single bid, pays the amount bid, and receives a share of the item equal to the amount bid divided by the sum of all bids.[1] [2] The present paper explores, experimentally, revenue and efficiency under these two auction formats with weakly and strongly budget constrained bidders.

The proportional auction was used in Russia's privatization of state-owned enterprises and with cryptocurrency sales, and has been suggested for selling limited space on the "cloud" (Teng and Magoulès, 2010; Tsai and Tsai, 2012). When Russia privatized its state-owned enterprises, given the limited wealth in the private sector, FPAs would have resulted in relatively low revenue with highly distorted wealth effects. To mitigate these issues, economists suggested using the PA auction (Boycko et al, 1994). More recently, the proportional auction has been used in cryptocurrency crowd-sales. For example, in 2017, *block.one*, a blockchain company, raised $4.2B by selling their cryptocurrency through a proportional auction (Howell et al., 2020).

The main goal of this paper is to conduct an experiment comparing revenue and efficiency of PA and FPA under different budget constraints in the context of a private value auction. In the experiment a single supplier sells a divisible good to n buyers. Each bidder is assigned a private value v, which is drawn from a uniform distribution U[0,100], with buyers having a common budget constraint w, the maximum they can bid. Theory predicts that if the budget is loose enough FPA will raise more revenue and have higher efficiency than PA. However, when the budget is tight enough PA can achieve more revenue and have greater efficiency than FPA, so it can be considered a good alternative to an FPA. This paper experimentally explores PA and FPA under a

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[1] The proportional auction is sometimes referred to as the *voucher auction* (Krishna, 2009) or the *proportional-share auction* (Dobzinski et al.,2012). In this paper, following Brooks and Du (2021), the term used is *proportional auction*.
[2] PA has some similarities to the widely studied Tullock contest (Tullock 1980). But they are quite different as in a Tullock contest the value of the item is publicly know and the same to everyone, with a winner-take-all outcome. Nevertheless, the analytic tools developed for studying the *incomplete information* Tullock auction can be applied here as well.

tight budget constraint, where PA is predicted to raise more revenue and be more efficient than FPA. As well as a looser budget constraint, where FPA is predicated to raise more revenue and have greater efficiency than PA. Both Are investigated within the context of an independent private value auction where, for simplicity, all bidder's face a common budget constraint.

The experimental outcomes are broadly consistent with the predictions of the theory: The FPA performed better in terms of revenue and efficiency under the looser budget constraint. But PA achieved higher revenue and efficiency under the tight budget constraint. Under both FPA and PA bidders tend to bid above the risk-neutral Nash equilibrium. Nevertheless, experimental outcomes were quite close to predicted outcomes.

The rest of the paper is organized as follows. Section 2 reviews the previous literature. Section 3 introduces the theoretical framework for analyzing the auctions in question. Section 4 describes the experimental design and hypotheses. The experimental procedures are outlined in section 5, with the experimental results reported in Section 6. Section 7 summarizes the outcomes reported.

2. Literature review

There have only been a few experimental studies of auctions that feature budget-constrained bidders. Pitchik and Schotter (1988) study two-stage sequential auctions in which two budget-constrained bidders can each strategically deplete the other bidder's budget during the first stage. Kotowski (2011, 2020) theoretically and experimentally studies the first price auction in which two bidders have private values and private budget constraints. Ausbel et al. (2017) experimentally study first and second price auctions in which the budget is endogenously set by a financial manager. In contrast the FPA without budget constraints has been widely investigated. The general finding is that the FPA typically achieves high efficiency and revenue in the absence of any kind of a budget constraint, reasonably close to equilibrium predictions (see Kagel and Levin. 2016, for a survey of the literature). However, subjects tend to consistently bid above the risk neutral Nash equilibrium generating higher revenue than predicted for risk neutral bidders.

3. Theoretical framework

A seller sells a divisible good to n bidders who want to buy at least a share of the commodity. All bidders are risk neutral and each bidder has a private value for the good. Let v_i be the value of the item for bidder i when the bidder receives the whole item. If the bidder receives a share $x \in$

$[0,1]$ of the good, their payoff for the share is $v_i x$. Values are independently drawn from a distribution $F(v)$. For simplicity, bidders have a common budget constraint w, which is the most they can spend on the item.

In FPA, each bidder submits a single bid less than or equal to their budget constraint w. The bidder with the highest bid wins the item and pays the amount bid. In case of a tie, one of the tied bidders is randomly selected as the winner. The FPA has a unique equilibrium when bidders have a common budget constraint (Milgrom, 2004). The equilibrium bid function has two parts, with bidders whose values are lower than the budget constraint biding according to the equilibrium of the FPA without a budget constraint. And bidders whose values are higher than the budget constraint submitting a bid equal to their budget constraint, w. The following proposition characterizes the equilibrium bid function.

Proposition 1. (Milgrom, 2004) Let $\hat{\beta}^{FPA}(v)$ be the unique equilibrium bid function in FPA for risk neutral bidders without a binding budget constraint. With a binding budget constraint w, the unique bid function $\beta^{FPA}(v,w)$ of FPA is

$$\beta^{FPA}(v,w) = \begin{cases} \hat{\beta}^{FPA}(v), & v < v^* \\ w \quad , & v \geq v^* \end{cases}$$

where v^* is the solution of the following equation

$$(v^* - w) \quad \frac{1 - F(v^*)^n}{n[1 - F(v^*)]} = (v^* - \hat{\beta}^{FPA}(v^*))F(v^*)^{n-1}$$

In the PA, each bidder submits a single bid for a share of the item and pays what she bid. Then each bidder receives a fraction of the item equal to their bid divided by the sum of all bids**.**

The PA also has a unique equilibrium.[3] Let $\beta^{PA}(v,w)$ denote the unique equilibrium in PA with a budget constraint w. $\beta^{PA}(v,w)$ is characterized by the following equation.

$$\beta^{PA}(v_i,w) = \text{argmax}_{0 \leq b_i \leq w}[\, \mathrm{E}_{v_{-i}} \left[\frac{b_i}{b_i + \sum_{-i} \beta^{PA}(v_{-i},w)}\right] v_i - b_i]$$

It is intractable to get an analytical solution for $\beta^{PA}(v,w)$ because the expected payoff does not have a closed form expression, as it has random variables (other bidders' values) in the

[3] See Wasser (2013) and Ewerhart (2014) who demonstrated that the *incomplete information* Tullock contest has a unique equilibrium. One can readily apply their results to determine equilibrium predictions for PA setup here (see Appendix A1).

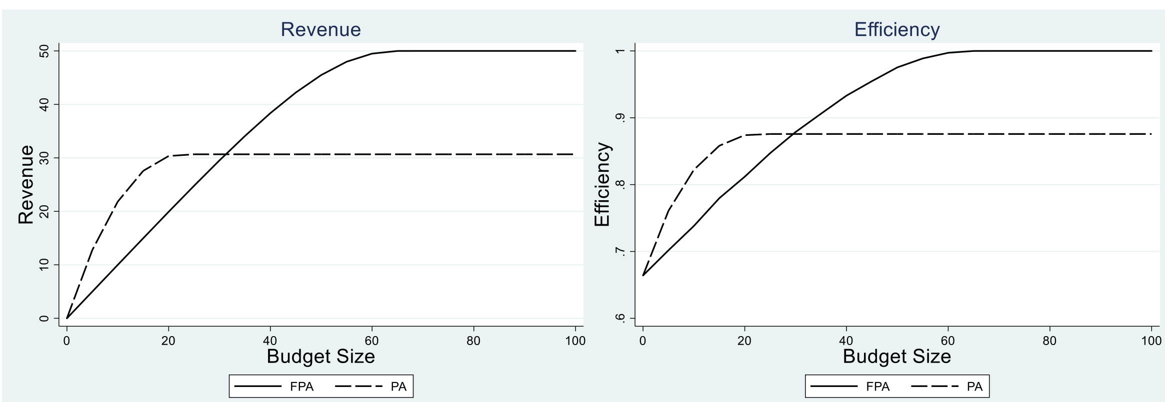
denominator. However, one can numerically approximate equilibrium bids.[4] The result is that in equilibrium, bids as always less than a bidder's value. Further, bidding above 1/4 v_i is dominated as the positive effect on increased earnings at this point is more than offset by the increase in the price for shares won.

Proposition 2. In PA, bidding more than 1/4 v_i is dominated by bidding 1/4 v_i, regardless of n and $F(v)$.[5]

This holds regardless of others' bids and the number of bidders. However, unlike FPA, where bidding above one's value is obviously dominated, bidding above 1/4 v_i is not obviously dominated.[6]

Figure 1 shows the expected efficiency[7] and revenue of FPA and PA across different budget levels, 3 bidders (the number of bidders in each auction), with values randomly drawn from U[0,100] (i.e., $F(v) = 1/100$), and a common budget constraint. (The parameter values used in the experiment.)

Figure 1. Efficiency and revenue predictions across different budget levels.



With no budget constraint, FPA achieves maximum efficiency, as it is an optimal auction. In contrast, with no budget constraint, PA has both lower efficiency and lower revenue than FPA,

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[4] The uniqueness of the equilibrium ensures that the numerical solution will be close to the theoretical equilibrium. A modified version of Wasser (2013)'s Matlab program was used to calculate the equilibrium bids. We thank him for kindly sharing his program.

[5] See Appendix A4 for proof of this proposition.

[6] For example, bidding above one's value in an FPA is obviously dominated, bidding above one's value in a second price auction is not (see Li, 2017 for a more general distinction between the two).

[7] Efficiency is defined as $S_{realized} / S_{max}$, where $S_{realized}$ is the realized surplus in an auction and S_{max} is the maximum possible surplus.

as PA is inherently inefficient on account of its allocation rule.[8] However, when the budget constraint is 30 or less, in equilibrium the PA has higher average efficiency than the FPA. That is, as the budget constraint gets tighter, FPA incurs a heavier efficiency loss than PA. So that in equilibrium, PA can achieve higher efficiency when bidders are severely budget constrained. These results indicate that when bidders have a strong enough budget constraint, PA can provide a good alternative to FPA. However, there is very little empirical/experimental evidence supporting this claim.

4. Experimental design and hypotheses.

The experiment employs a 2 by 2 design with two auction formats (FPA, PA) and two budget constraints: $w = 50$ for the weak budget constraint and $w = 20$ for the tight budget constraint. Each auction, three bidders ($n = 3$)[9] with each bidder is assigned a private value v that is independently drawn from a uniform distribution on (0, 1, 2, …, 100). As a simplification all bidders have the same budget constraint.[10]

Table 1 shows equilibrium predictions for efficiency and revenue in the two auctions under the two budget levels: On average PA is predicted to have higher revenue and efficiency than FPA under the tight (w=20) budget constraint. In contrast, under the looser budget constraint (w=50) average revenue and efficiency is predicted to be higher under FPA than PA. These are the main hypothesis investigated here.

Table 1. Equilibrium predictions (average values)*

		FPA	PA
w=20	Efficiency	81.1%	87.4%
	Revenue	19.9	30.4
w=50	Efficiency	97.5%	87.6%
	Revenue	45.5	31.0

* Based on realized bidder values. . Efficiency is defined as $S_{realized}$ /S_{max}, where $S_{realized}$ is the realized surplus in auctions and S_{max} is the maximum possible surplus.

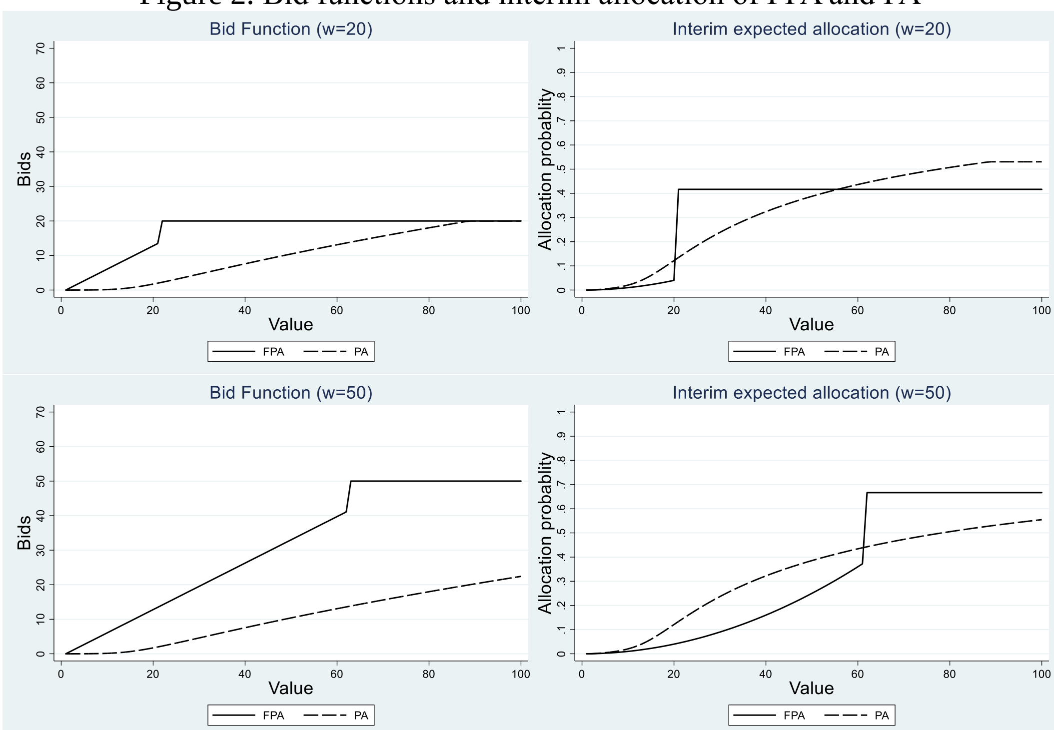
[8] Under the current setting, revenue is closely tied to efficiency. Ausbel and Crampton (1999) show that under any constant marginal utility environment without a reserve price, revenues are maximized by allocating the item efficiently.

[9] Appendix A2 discusses the comparative statics of the two auctions with different number of bidders.

[10] Without this, outcomes would be dependent on the distribution of budget constraints as well as the distribution of valuations and the corresponding bids.

The intuition behind the results in Table 1 are shown in Figure 2, which includes bid functions and interim expected allocations.[11] Under the strong budget constraint (w = 20), FPA randomly allocates items to bidders with values between 22 and 100, so that high value bidders have a smaller chance of getting the item (see the interim expected allocations).[12] In contrast, under the weak budget constraint (w = 50) there is less pooling of high and low value bidders, increasing the chance that a high value bidder will get the item. Consequently, for PA to have higher efficiency than FPA depends on bidders reasonably closely approximating equilibrium outcomes under both mechanisms

Figure 2. Bid functions and interim allocation of FPA and PA



5. Experimental procedures

[11] Interim expected allocation refers to the expected share, or probability, of getting an item given one's own value when the others' values are unknown.

[12]Pooling occurs for bidders with values between 22-100, not including bidders with value 20. A bidder with value 20 would want to bid less than this in equilibrium.

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A total of 6 sessions were run, 3 sessions for FPA and 3 for PA. Between 15 and 21 subjects in each session.[13] Subjects participated in 10 auctions under w=20, followed by 10 auctions with w=50.[14] In each auction, subjects were assigned integer values randomly drawn from [0, 100] with subjects randomly rematched into different groups in each auction. In FPA, each bidder could submit an integer bid up to the budget constraint w, and the bidder with the high bid won the item and paid the amount bid. In the case of ties, one of highest bidders was randomly selected to be the winner. Similarly, in PA, bidders could submit integer bids up to the budget constraint. After all bids were submitted, each bidder paid their bid and received a fraction of the item equal to their bid divided by all bids. [15] Subjects were provided with an on-screen calculator for PA sessions where they could enter potential bids for their opponent along with their bid, to easily calculate their expected payoff for different possible bids.[16]

Subjects participated in three practice rounds, with all bidders assigned the same values, bidding against two computerized bidders. The same computerized bidders were assigned to all subjects to ensure that everyone had the same experience. Within each session a new set of random values were drawn. The change in the size of the budget constraint after the first 10 periods was not announced in advance.

In each auction, subjects were given 1 min (1.5 min for the first three paid auctions) to make decisions. Subjects were provided with starting capital balances of 100 experimental currency units (ECUs) with earnings added to this and losses subtracted from it. Bidders were told they could lose money, but no restrictions were imposed on bids to prevent this. They were also told that if their cash balance went to zero or negative, they would be bankrupt and no longer able to bid. This never happened.

Following each auction, feedback was provided in the form of a table reporting values, bids, allocations, and earnings of all bidders. All values and earnings were denominated in ECUs. Final earnings were converted into dollars at the rate of 20ECUs=$1. Earnings averaged $19.82 per subject in FPA and $19.50 in PA, with sessions lasting 1.5 hours on average.

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[13] FPA sessions had 21 subjects in each session, while PA sessions had 18,21,15 subjects in each session. A different set of subjects participated in each experimental session.
[14] Two additional FPA and one additional PA were run in the reverse order, with the results essentially the same. These are reported in the online Appendix.
[15] There were two auctions in which all bidders submitted a 0 bid, with these bidders receiving nothing and paying nothing.
[16] This was provided for PA because of the more complicated nature of determining potential payoffs.

The experiment was run in the Ohio State University Experimental Economics Laboratory between Mar 2019 and Sep 2019. Most of the subjects who participated in the experiment were undergraduate students drawn from all disciplines, and all were recruited through ORSEE (Greiner, 2004). Each subject participated in no more than one experimental session. The experiment was computerized and programmed using z-Tree (Fischbacher, 2007).

6. Experimental results

6.1 Efficiency and revenue

Table 2. Predicted and realized efficiency/revenue

	Budget	Predicted		Realized	
		FPA	PA	FPA	PA
Effi-ciency	w=20	81.6% (0.8%)	87.7% (0.4%)	81.2% (1.7%)	87.0% (0.6%)
	w=50	98.1% (0.3%)	87.8% (0.4%)	96.2% (0.6%)	90.2% (0.6%)
Revenue	w=20	19.9 (0.07)	30.1 (0.92)	19.8 (0.10)	29.7 (0.91)
	w=50	45.2 (0.65)	30.4 (0.95)	46.6 (0.55)	37.8 (1.56)

* Standard errors of the mean are in parentheses.
* Predicted outcomes were calculated based on equilibrium predictions for the random values drawn.

Table 2 shows predicted and realized average efficiency and revenue under the two budget constraints. Realized efficiencies were broadly consistent with the theoretical predictions. The main hypothesis clearly holds: PA achieved significantly higher efficiency (87.0%) than FPA (81.2%) under w=20 (Mann-Whitney test, $p < 0.05$ using session averages as the unit of observation), and FPA achieved significantly higher efficiency (96.2%) than PA (90.2%) under w=50 ($p < 0.05$). Realized efficiencies for FPA and PA were also quite close to the theoretical predictions - all within 3 percent points of the predicted efficiencies.[17]

Realized revenues were also broadly consistent with the theoretical predictions. As the main hypothesis predicted, there was a revenue reversal between FPA and PA under the strong and weak budget constraints: Under w=20, PA raised significantly more revenue (29.7 ECUs) than FPA (19.8 ECUs, $p < 0.05$), while under w=50, FPA raised significantly more revenue (46.6 ECUs)

[17] Regressions with dummy variables for the two auction formats and the two budget constraints, with clustering at the session level yield essentially the same as the Mann Whitney test. (see online Appendix A3).

than PA (37.8 ECUs, $p < 0.05$). Realized revenues were, for the most part, quite close to the theoretical predictions, although PA achieved noticeably higher revenue (37.8) than predicted (30.4) under w=50. As shown below, this resulted from high-value bidders bidding well above 1/4 v_i benchmark in PA with w=50 (see Figure 6 below). Finally, note that average revenue under PA with w=20 was greater than 20 ECUs, a level not attainable under FPA.

Result 1: The main hypotheses were well supported: PA achieved significantly higher efficiency and revenue than FPA under the tight (w=20) budget constraint, while the opposite held under the weaker budget constraint (w=50). Realized outcomes were quite close to equilibrium predictions, except that PA achieved noticeably higher revenue than predicted under w=50. The absence of a strict ranking in revenue between the two auction formats is consistent with the idea that when bidders budget limit their ability to buy the whole item, sellers could well be better off using a PA as opposed to an FPA auction. And the PA would yield more efficient allocations than FPA.

6.2 Bidding behavior

6.2.1. First price auction

Figures 3 shows predicted and realized interim allocations for the FPA auctions. Interim allocations show how frequently bidders with different values can expect to get the item. For example, under FPA with w = 20, in equilibrium a bidder with value 22 was expected to win the item 41.6% of the time, and a bidder with value 100 was expected to win the item at the same rate, contributing to the low predicted efficiency under w=20. Realized allocations were consistent with these predictions as bidders with values between 22-100 won with about the same frequency; e.g., bidders with values between 25-35 won the item 44.0% of the time, while bidders with values between 90-100 were high bidders 45.4% of the time.

Under w=50, in equilibrium FPA was predicted to allocate the item mostly to high-value bidders. For example, there was only a 4.8% chance that a bidder with value 22 would win the item compared to 66.7% chance that a bidder with value 100 would get the item. Realized allocations were consistent with these predictions as high-value bidders won much more frequently than low-value bidders. For example, bidders with values between 25-35 won the item 11.0% of the time, while bidders with values 90-100 got the item 61.9% of the time.

Figure 3. Predicted and realized interim allocation in FPA

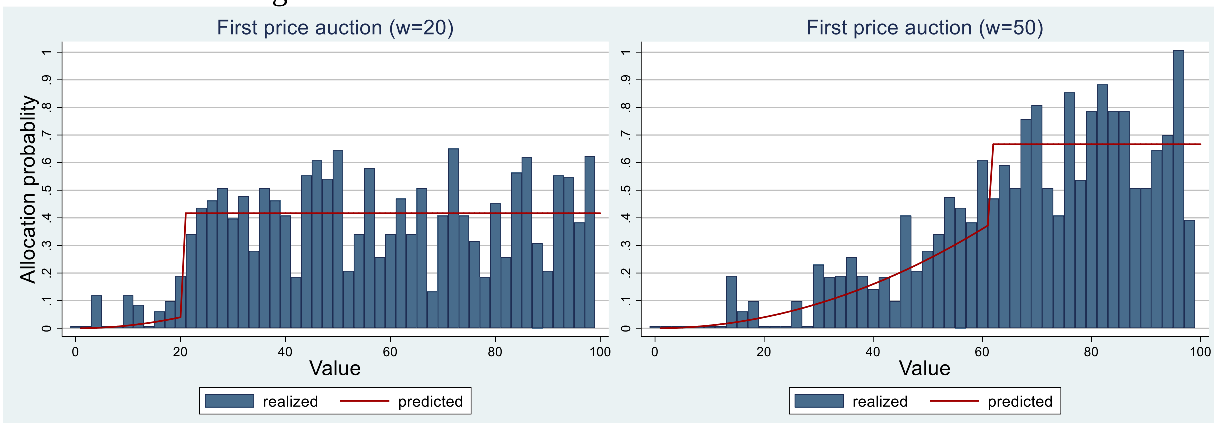

Figure 4. Bid plots in the first price auction

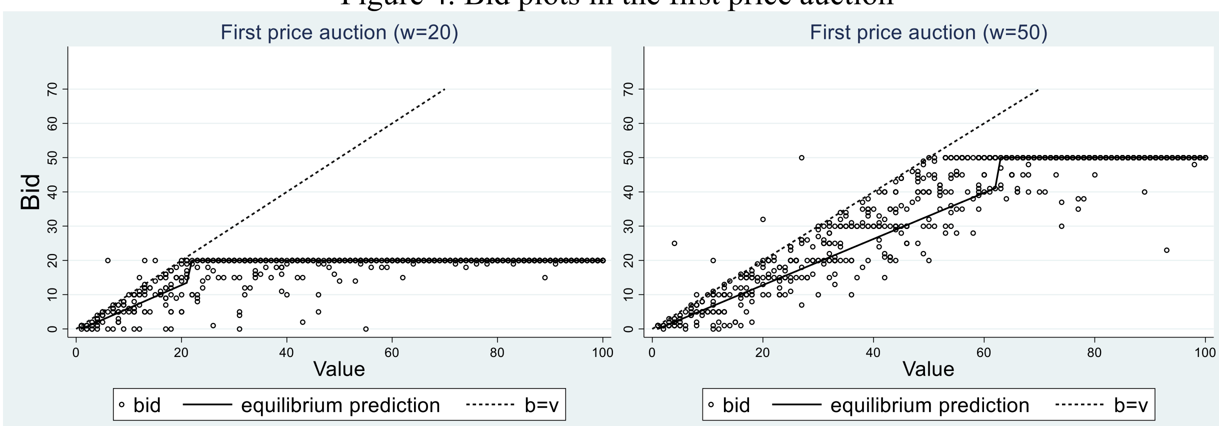

Figure 4 shows submitted bids in FPA sessions under *w*=20 and *w*=50. The solid lines are equilibrium bid functions. with the dashed lines representing bids equal to value. Under *w*=20, budget constrained bidders were expected to pool at the budget constraint, which they did 87.3% of the time. Bidders with values below 22 were expected to bid two-thirds of their value (the risk neutral Nash equilibrium bid) But as is typical of FPAs, 44.2% of these bids were more than 1 ECU above this benchmark.

Under *w*=50, bidders with values equal to or greater than 63 were predicted to pool at the budget constraint, with lower valued bidders predicted to bid two-thirds of their values under the risk neutral Nash equilibrium (RNNE). 88.6% of bidders with values greater than 63 bid the budget constraint, with 11.4% bidding below it. 56% of unconstrained bids were more than 1ECU above the RNNE benchmark. Further, a notable portion (49.3%) of bidders with values between 51 and 62 pooled at the budget constraint. Bidding above the RNNE for unconstrained bidders is consistent with previous experimental studies of first price private value auctions, without budget constraints. Indeed, bidding above the RNNE is the most common outcome in single-unit FPAs (Kagel, 1995).[18] However, unlike previous studies, realized efficiency and revenue only deviated slightly from theoretical predictions, with high-value budget constrained bidders pooling at the budget constraint.[19]

6.2.2 Proportional auction

Figure 5. Predicted and realized interim allocations in PA

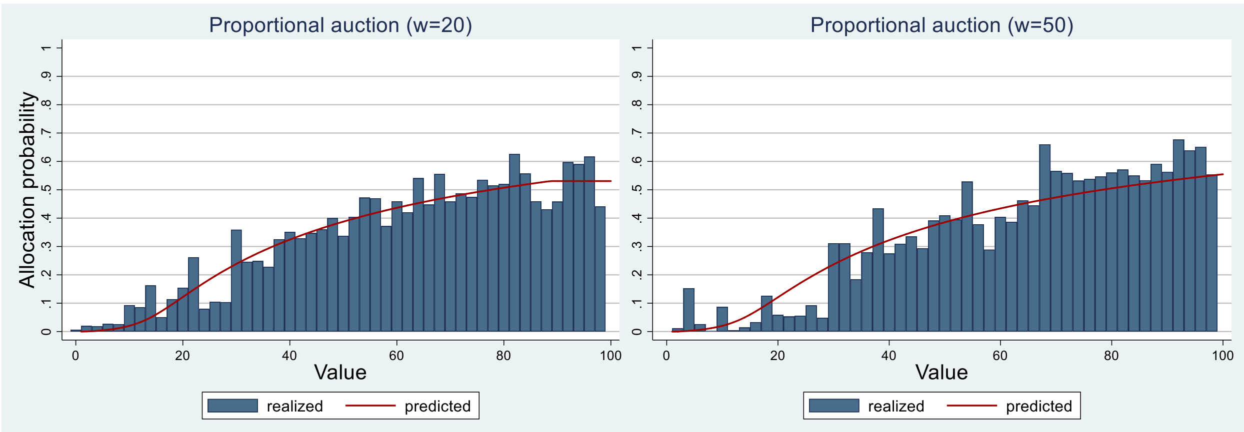

Figure 5 shows predicted and realized interim allocations under PA. Under w=20, interim allocations were quite close to predicted levels. Under w=50, bidders with values below 30 received fewer shares than predicted, while bidders with values above 70 received more shares than predicted, with these deviations resulting in PA achieving higher efficiency than predicted under w=50 (90.2% vs 87.8%, see table 2, p-value < 0.01).

[18] Overbidding behavior in FPA could be rationalized by factors such as risk aversion, the joy of winning, and anticipated regret. See Cox et al. (1988), Kagel (1995), Goeree et al. (2002), Filiz-Ozbay and Ozbay (2007), and Kagel and Levin (2016) for more details.

[19] Bidding more than one's value was exceedingly rare: 0.63% under w=20 and 0.79% under w=50.

Figure 6. Bid plots in the proportional auction.

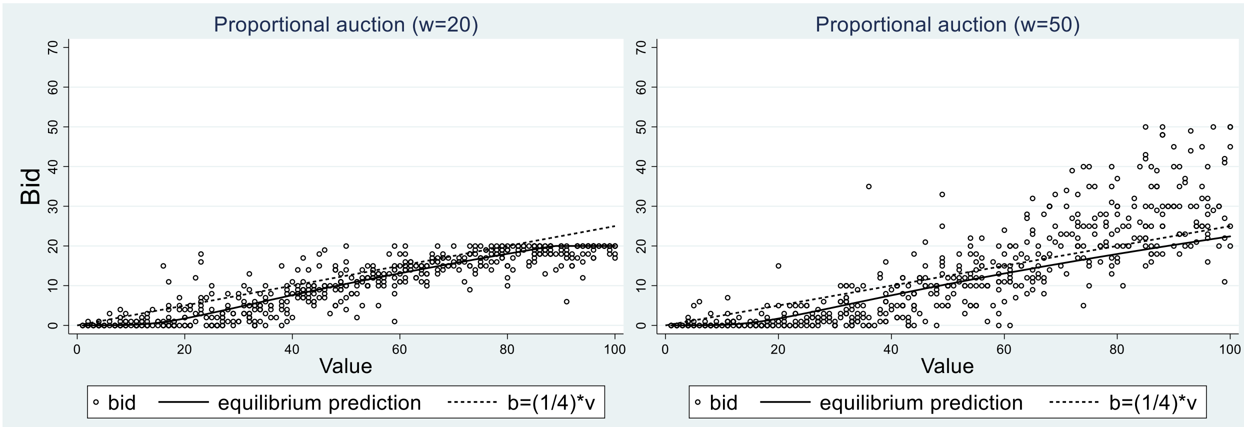

Figure 6 plots PA bids under the two budget constraints. The solid lines are equilibrium bids, with the dashed lines at 1/4 v_i , the cutoff point for undominated bids (Proposition 2). Under w=20, 55.9% of all bids were within 1ECU of the theoretical prediction, with 24.8% (19.3%) bidding below (above) equilibrium. The net effect was that realized revenue and efficacy were quite close to predicted levels (see Table 2).

Under w=50, realized bids are more scattered relative to predicted bids than with w=20. Bidders with values below 40 bid close to predicted levels: 57.4% within 1ECU of equilibrium, with 29% (13.5%) bidding below (above) equilibrium. However, bidders with values greater than 40 showed substantial bidding above equilibrium (58.3%), with 53.1% bidding above the 1/4 v_i benchmark for dominated bids. The preponderance of bidding above the benchmark for dominated bids largely accounted for the higher than predicted revenue under PA (37.8 ECUS versus 30.4; 24% higher). While high value bidders typically bid above equilibrium, and low value bidders did not, high value bidders got larger shares than predicted, contributing to the higher than predicted efficiency with w=50 (90.2% actual compared to 87.8% predicted).

As for the frequency and level of dominated bids, high value bidders (value >40) did not lose money much more often with dominated (13.3%) compared to undominated bids (9.8%). In contrast, bidders with values below 40 lost money 66.7% of the time when submitting a dominated bid compared to 24.4% of the time with an undominated bid. So that losses from dominated bids were more frequent for low value bidders, but not high value bidders. Imposing greater discipline against dominated bids for low value compared to high value bidders.[20] Thus, the low-value bidders were disciplined by the loss.

Result 2: Bidding in FPAs was largely consistent with equilibrium predictions under both budget constraints, except for the tendency to bid above the RNNE, widely reported in FPAs without budget constraints. Bidding behavior in PA was close to the equilibrium prediction under w=20. However, with w = 50, a substantial number of high value bidders submitted dominated bids. The latter no doubt can be accounted for by the fact that bidding above the 1/4 v_i benchmark is not obviously dominated. Further, the opportunity cost of dominated bids was relatively small: Subjects who bid above ¼ v_i lost on average 3.19 ECUs with w=50, while often winning and making positive profits.[21]

6. 2. 4: Comparing Realized and Predicted Outcomes to Zero Intelligence Bidding

One thought that occurs is that with the tight budget constraint (w = 20) employed here, outcomes under both PA and FPA were so constrained, that there was little room for differences between predicted and actual outcomes. One way to check this is to compare predicted and actual behavior to zero intelligence (ZI) bidders (Gode and Sunder, 1993), which have proved to provide reasonable approximations to actual outcomes in computerized double auction markets. The implication being that the structure of double auctions markets accounts for much of the consistency between actual and predicted outcomes there. The question posed here is whether the same could be said for PA compared to FPA under the tight budget constraint.

In the simulations ZI bidders were restricted to bidding randomly between 0 and their private value, and pooling at the budget constraint for bids exceeding their budget constraint. Table 3 below reports average efficiency and revenue under the w = 20 and 50 budget constraints for ZI traders compared to both equilibrium and experimental outcomes. [22]

[20] All of this is, of course, based on the fact that bidding above the ¼ v benchmark is far from obvious.
[21] With w=20 average loses from dominated bids were even less: .82 ECUs ($0.04). +
[22] Results are based on 2100 random draws for each valuation used in the experimental sessions.

Table 6: ZI Simulations versus Equilibrium and Realized Experimental Outcomes

	Budget	Predicted		Realized		ZI Simulations	
		FPA	PA	FPA	PA	FPA	PA
Efficiency	*w*=20	81.6% (0.8%)	87.7% (0.4%)	81.2% (1.7%)	87.0% (0.6%)	85.2% (0.6%)	77.0% 0.4%)
	w=50	98.1% (0.3%)	87.8% (0.4%)	96.2% (0.6%)	90.2% (0.6%)	89.4% (0.5%)	80.9% (0.4%)
Revenue	*w*=20	19.9 (0.07)	30.1 (0.92)	19.8 (0.10)	29.7 (0.91)	19.5 (0.10)	41.9 (0.42)
	w=50	45.2 (0.65)	30.4 (0.95)	46.6 (0.55)	37.8 (1.56)	38.4 (0.43)	68.1 (0.98)

* Standard errors of the mean are in parentheses.
* Predicted outcomes were calculated based on equilibrium predictions for the random values drawn.

Several things are worth noticing. In the ZI simulations efficiency in PA is lower than in FPA for w = 20, contrary to both the experimental and equilibrium outcomes. So indeed there was room for experimental outcomes to differ from predicted outcomes, in spite of the tight budget constraint employed. There were other results of note as well: Under FPA, with w = 20, ZI efficiency was *greater* than both predicated and realized efficiency, which at first glance seems puzzling. This can be explained as follows: First, in FPA with w = 20, in equilibrium bidders with values between 22 and 100 pool at 20. But with ZI bidders randomly bidding between 0 and their valuations, the chances that a bidder with value 22 will pool at 20 are smaller than a bidder with value 80 pooling at 20. So even if the latter bids proportionately less, their bid is more likely to be constrained at 20 than the lower valued bidder. Second, under both budget constraints, ZI revenue is substantially higher in PA than either predicted or actual revenue. This suggests that in the experiment bidders moderated the extent to which they bid above the ¼ v benchmark, quite possibly on account of losses suffered in earlier periods. In contrast, ZI bidders don't learn to moderate their bidding on account of previous losses.[23]

7. Summary and Conclusions

This paper experimentally investigates proportional auctions (PA) comparing them to first price auctions (FPA) under both weak and strong budget constraints. Under the strong/tight budget constraint, as predicted both revenue and efficiency were higher under PA than FPA. In contrast,

[23] In the first 5 periods with w = 50, the frequency of bidding above the ¼ v benchmark was 57.1% compared to 49.6% in the last 5 periods.

with the looser budget constraint, both revenue and efficiency were predicted to be higher under FPA than PA. With corresponding outcomes in the experimental data. These results, in addition to exploring bidding under PA, provide insight into why the PA format has been used in Russia's privatization of state assets, as well as cryptocurrency sales, market, where bidders have strong budget constraints compared to the quantities being sold.

The current study can be extended in several ways. The most obvious way would be to explore the impact of heterogenous budget constraints as well as the effect of increased numbers of bidders. Further from the present study would be to investigate PA compared to uniform price auctions, where on account of tight budget constraints, bidders demand only a portion of the bid on.

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